

Stability Analysis of Slender Passageway of A UW's Campus Center

Sabbir SIDDIQUE

Consulting Engineer

Dhaka, Bangladesh

sabbirsiddique@yahoo.com



Sabbir Siddique, born 1961, received his B.Sc engineering degree from the Rajshahi Engineering College, and M.Sc. from Bangladesh University of Engineering and Technology. He worked for different companies in Bangladesh before becoming a freelance professional. His main area of interest is related to earthquake analysis, structural risk assessment and optimization.

Summary:

Stability represents a fundamental problem in structural engineering which must be investigated to ensure the safety of structures against collapse. The importance of the subject is evident from the history of structural collapses caused by neglecting the stability aspects in design.

In the Academic Building of A UW, there is a long passage of three storied high supported on thin columns that supports a garden at the top. The passage columns need to be thin in size and not allowed to connect in the longitudinal direction. Stability of the columns in the unconnected direction was in question. During finalization of construction design, stability of the structure was mathematically checked by linear buckling analysis. The aim was to find the buckling modes and corresponding critical buckling loads. This paper represents the finding of the stability analysis and discusses fundamental issues of stability of the building structures.

Keywords: Instability, Euler critical load, buckling, stability, eigenvalue.

A person, who never made a mistake, never tried anything new. **-Albert Einstein**

1 Historical Background

Perhaps the collapse of the Tacoma Narrow Bridge in 1940 was the most important event of structural collapses caused by neglecting the stability aspects of design. Many other such events from the early history helped to understand stability aspect of structures today. History of stability analysis began with Leonhard Euler, a Swiss mathematician and physicist who derived a formula in 1757 that gives the maximum axial load that a long, slender, ideal column can carry without buckling.

2 Introduction

Structural stability can be broadly defined as *the power to recover equilibrium*. Buckling of column is very common among the different forms of instability.

Euler buckling is possibly the most widely encountered and studied buckling mode, both in theory and in practice. For the simplest case of a column under compressive axial load, the equation for Euler's buckling load can be obtained analytically as,

$$P_{CR} = \frac{\pi^2 EI}{L_e^2} \dots\dots\dots \text{Equation 1}$$

in which L_e is the effective length of a column, P is the axial force and π is a constant equal to 3.14159...



3 Testing Stability

The stability of a structure in particular, can be tested (experimentally or analytically) by observing how it reacts when external disturbances are applied. Three possible outcomes are sketched in Figure 1 to Figure 3.

Stable For *all* admissible perturbations, the structure either returns to the tested configuration or executes bounded oscillations about it. If so, the equilibrium is called *stable*.



Figure 1 Stable equilibrium

Unstable If for at least *one* admissible perturbation the structure moves to (decays to, or oscillates about) another configuration, or “takes off” in an unbounded motion, the equilibrium is *unstable*.

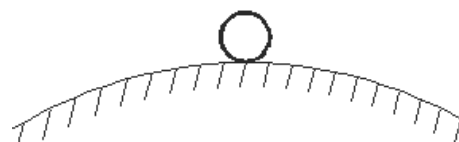


Figure 2 Unstable equilibrium

Neutral The transition from stable to unstable occurs at a value λ_{cr} , which is called the *critical load factor*. At the critical load factor, the structure is said to be in *neutral equilibrium*.



Figure 3 Neutral equilibrium

The quantitative determination of this transition is a key objective of the stability analysis.

4 Mathematical Formulation

The buckling analysis problem can be narrowed to an eigenvalue analysis problem.

$[K] + \lambda_i [K_G] = 0$ where, λ_i eigenvalue (critical load factor)

From the eigenvalue analysis, eigenvalues and mode shapes are obtained, which correspond to critical load factors and buckling shapes respectively.

5 Analysis & Result

Buckling analysis of the colonnade was done following eigenvalue method and linear elastic finite elements analysis to calculate the buckling load of the structure. First buckling mode shape is shown in Figure 4. Five buckling mode shapes were extracted. Critical loads for different modes can be seen in Table 1.

Table 1 Critical loads for different buckling modes

BUCKLING ANALYSIS		
Mode	Eigenvalue	Tolerance
1	07,016326	5,0635e-016
2	20,399530	1,0554e-013
3	20,501620	3,7379e-013
4	21,491867	2,6673e-011
5	24,653715	2,0772e-007

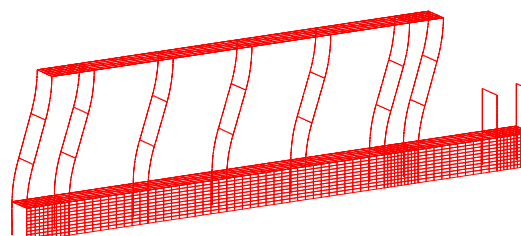


Figure 4 Buckling mode shape – Mode 1

For instability of the structure only the first buckling load and mode shape has practical significance. Buckling in the higher modes generally is of little importance because the system will have failed when the load exceeds the lowest critical loads.

For the structure under investigation, it can be seen that instability from buckling of columns can be expected if the load on the top of the columns is 7,01 times more than the loading expected in the structure during its service life.