Serviceability Performance of Material Efficient Concrete Structures

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Summary

This paper deals with some basic features of optimized reinforced concrete structures. For orthogonal mesh reinforced panels with given boundary conditions and given outer geometry, the theoretical criterion for a material efficient (optimized) structural layout is established. The criterion is based on a lower bound plasticity design approach. Subsequently, by using the principle of minimum complementary elastic energy, it is shown that for optimized panels, there is a close relationship between the stress distribution in the fully cracked elastic state and the stress distribution used in the ultimate limit state. It is discussed how this finding may be used for serviceability analysis of optimized structures.

Keywords: Optimized concrete structures, fully cracked elastic state, serviceability performance.

1. Introduction

Material efficient structures, also called optimized structures, can be defined as structures designed in such a way, that the material consumption is minimized. The most efficient way to obtain optimized structures is to use plasticity based design methods. In the Eurocode 2 [1], a wide range of plasticity based models have been adopted for design of concrete structures. For instance, the yield conditions developed by Nielsen [2] have been implemented to facilitate plastic lower bound design of mesh reinforced panels. Another lower bound method mentioned in Eurocode 2 is the Strut and Tie method. When these methods are combined with numerical optimization techniques, it is possible to fully optimize and obtain material efficient concrete structures. Since the plasticity based design methods only deal with the ultimate limit state, separate analyses of the serviceability performance of the structures must be conducted. Traditionally, structural engineers have the presumption that structures, which are optimized by plasticity methods, will exhibit poor serviceability behaviour - for instance severe cracking or significant stress redistributions. This is, however, not the case as demonstrated in this paper.

It is shown that for structures which are fully optimized, there is a close relationship (affinity) between the stress distribution in the ultimate limit state and the stress distribution in the fully cracked elastic state. Hence, none or minimum stress redistribution will take place when the load is increased from the fully cracked elastic state to the ultimate limit state.

The paper will only deal with orthogonal mesh reinforced panel structures. For other types of structures, such as two-way spanning slabs and panels designed as Stringer-Panel systems, similar investigations have been carried out in ref. [3].

2. Optimization criteria

An orthogonal mesh reinforced panel is considered, Fig. 1. For any given admissible stress distribution [$N_{x,u}$(x,y); $N_{y,u}$(x,y); H_u(x,y)], plasticity based design formulae may be used to calculate the necessary reinforcement amounts A_{sx} and A_{sy} (area per. unit length) as well as the necessary concrete thickness at any point on the structure.

As a measurement of the material efficiency of a given structural layout, the weighted material consumption is defined as: $V_{weighted} = V_s + f_c/f_y V_c$. Here V_s is the reinforcement volume and V_c is the

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concrete volume. *The stress distribution* to be used in order to obtain an optimized structural layout has to fulfil Eq. (1).

$$\iint \{A_{sx}(x,y) + A_{sy}(x,y)\} dxdy + \frac{f_c}{f_y} \iint t(x,y) dxdy = \min \quad \Leftrightarrow \quad V_{weighted} = \min \tag{1}$$

If it is required, in the fully cracked elastic state, that $\sigma_{sx} = \sigma_{sy} = \alpha f_y$ and $\sigma_c = \alpha f_c$ throughout the structure (α being the proportionality factor between the ultimate load and the service load), then the stress distribution and the structural layout must be such, that the complementary elastic energy Π is minimised. The design criterion for this case is given in Eq. (2).

$$\frac{f_y^2}{E_s} \iint \{A_{sx}(x,y) + A_{sy}(x,y)\} dxdy + \frac{f_c^2}{E_c} \iint t(x,y) dxdy = \min \quad \Leftrightarrow \quad \Pi = \min \tag{2}$$

In the paper, it will be shown that practically, the plasticity based criterion (1) and the elastic criterion (2) will lead to identical structural layout. The implication of this result is, that for an optimized (material efficient) structure, there is an affinity between the stress distribution in the fully cracked elastic state and the stress distribution used in the ultimate limit state design. This means that the ultimate stress distribution can be downscaled to the service load level and used as the basis for serviceability analysis.

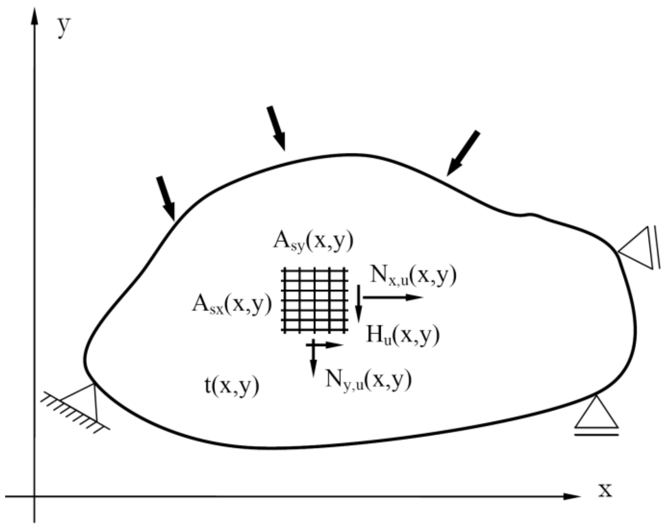

Fig. 1: Concrete panel with orthogonal mesh reinforcement.

3. References

[1] European Committee for Standardization, "*EN 1992-1-1, Eurocode 2: Design of Concrete Structures - Part 1-1: General Rules and Rules for Buildings*", 2005.

[2] NIELSEN M. P., "*Om jernbetonskivers styrke*" (*English: On the strength of reinforced concrete dics*), doctoral dissertation, Polyteknisk Forlag, Copenhagen, 1969, 254 pp.

[3] HOANG L. C., "On the elastic behaviour of plastic optimised concrete structures", *Proceedings of the Danish Society for Structural Science and Engineering*, Vol. 74, No.2 & 3, 2003, pp. 21 – 90.